Sub-Exposure Times and Signal-to-Noise Considerations

Revised: February 3, 2010

Introduction

Present guidelines for sub-exposure duration concentrate on reducing random noise. Random noise is the combination of Poisson distribution noise from the arriving photons and thermal noise as generated in the CCD chip preamplifier. The latter is often referred to as Johnson noise or shot noise. However imperfect detectors and outside influences conspire to further degrade the signal-to-noise ratio (SNR) by the introduction of this non-random noise. Examples of this type of noise are hot and cold pixels as well as cosmic rays and other transient artifacts. An approach is described which addresses both types of noise with minimal overall compromises.

Discussion

To address random noise such as that arising from, conventional Signal-to-Noise equations are normally used. This equation, whenever it is discussed, is quite complex and had many variables. Indeed, a complete SNR equation is still a subject of debate. What follows is a greatly simplified, but nevertheless I believe valid approach to determining sub-exposure times.

(1) $$SNR = \sqrt{N}\frac{E_{obj}t}{\sqrt{E_{obj}t + E_{sky}t + N_d + R_{on}^2}}$$

Where

- N = number of sub-exposures and assumes a mean combine.
- E_{obj} = Object flux in electrons per minute
- E_{sky} = Sky background flux in electrons per minute
- t = Exposure time in minutes
- N_d = Number of dark current electrons
- R_{on} = Readout noise in electrons

Equation 1 above identifies the key variables to determine the detected SNR of an object, assuming all the noise is random. For the bright source case, where $E_{sky}t \ll E_{obj}t$, the SNR becomes the somewhat trivial

$$SNR = \sqrt{NE_{obj}t}$$

But our interest is in the faint source case, where $E_{sky}t \gg E_{obj}t$. For this case, I will assume a sufficient number of dark frames have been taken and combined to reduce the dark current noise contribution. Now, Equation 1 becomes

(2) $$SNR = \sqrt{N}\frac{E_{obj}t}{\sqrt{E_{sky}t + R_{on}^2}}$$

So, how is sub-exposure time best determined? For a given background, we begin by determining how long an exposure is required to overwhelm the readout noise of the CCD by the sky background. Here we try to make the first term of the denominator dominate the second term, readout noise. This exposure time, t_{ORN}, is given by:

$$(3) \qquad t_{ORN} = \frac{R_{on}^2}{[(1+p)^2 - 1]E_{sky}}$$

Where p is the percentage increase in noise we will tolerate as a contribution from the camera readout electronics. For example, assuming we will tolerate a 5% increase in noise from the readout electronics, then the minimum exposure is given by:

$$(4) \qquad t_{ORN} = \frac{9.76 R_{on}^2}{E_{sky}}$$

E_{sky} can be determined by taking a few minute exposure of duration t_{test} and then measuring the average background of the resulting image and calling this $ADU_{background}$. E_{sky} is then calculated based on this data as follows:

$$(5) \qquad E_{sky} = \frac{(ADU_{background} - 100)g}{t_{test}}$$

100 should be subtracted from the background ADU if your camera control program adds a 100 count pedestal to prevent negative numbers. Most programs do.

For example, assume your camera has a readout noise, R_{on}, of 10e and a gain of 1.4 e/ADU. Further, a 5 minute test exposure with your luminance filter gives you a dark-subtracted background count of 400 ADU. Thus, your background sky flux is given by:

$$E_{sky} = \frac{(400 - 100)x1.4}{5} = 84e/\min.$$

Let's assume you want to allow readout noise to be no more than 5% of your total noise, p = .05 in equation 3. The minimum exposure time, t_{ORN} you must expose for, is given by

$$(6) \qquad t_{ORN} \geq \frac{9.76 R_{on}^2}{84e/\min.} = \frac{976}{84} = 11.6\min.$$

With this scenario, readout noise will comprise 5% of the total noise if your exposure time is 11.6 minutes.

It was mentioned that there is more than normal random noise involved. Other noise sources such as hot and cold pixels, CCD pattern noise, cosmic rays and other artifacts are non-Gaussian or non-random noise sources. These kinds of noise are best reduced with a combination of dithering and more sophisticated data combination approaches. Since these approaches are statistical in nature, it is appropriate that we get more samples, if we are to begin to have a statistically significant number of data points for each pixel after dithering and aligning.

Assuming our total exposure time is limited, one way to get more data points is with shorter exposures. Examining Equation 2, we see that the SNR increases by the square root of the number of frames. Let's consider this situation:

(8) $$t = \frac{t_{ORN}}{f},$$

where f is the desired reduction factor of exposure time.

We can determine the trade-off between exposures of t_{ORN} and fractions thereof by computing the SNR for both cases and setting them equal as follows:

(9) $$SNR(N_2) = SNR(N_1)$$

N_1 is the number of exposures at t_{ORN} and
N_2 is the resultant number of shorter exposures that is required to get the same SNR.

Using equation (2) for both sides of the equation,

(10) $$\sqrt{N_2}\,\frac{E_{obj}t_{ORN}}{f\sqrt{\frac{E_{sky}t_{ORN}}{f} + R_{on}^2}} = \sqrt{N_1}\,\frac{E_{obj}t_{ORN}}{\sqrt{E_{sky}t_{ORN} + R_{on}^2}}$$

And letting $$t_{ORN} = \frac{R_{on}}{[(1+p)^2 - 1]E_{sky}},$$

We obtain the following relationship for N_2:

(11) $$N_2 = \frac{f + f^2[(1+p)^2 - 1]}{(1+p)^2} N_1$$

Where

p is the allowable increase in noise due to readout noise
f is the t_{ORN} reduction factor.

We can use a similar approach to assess SNR loss as a function of dividing our exposure into an increasingly large number of shorter exposures. Using an approach similar to that shown above, we can develop the following relationship:

$$(12) \qquad SNR_f = SNR_1 \sqrt{\frac{(1+p)^2}{1+f[(1+p)^2-1]}}$$

Where

SNR_f is the SNR obtained with exposure time of t_{ORN}/f
SNR_1 is the SNR obtained with exposure time of t_{ORN}
p is the allowable increase in noise due to readout noise
f is the t_{ORN} reduction factor.

Equations (11) and (12) are very significant. Note that readout noise and exposure time are not direct factors. They are parametric factors that determine t_{ORN} and t_{ORN} alone. Of course, t_{ORN} is considerably longer at a dark site than at a rural suburban location.

We can develop a number of interesting cases to investigate this relationship. Assume t_{ORN} is 10 minutes and assume we want to see how shorter exposures compare to 50 minutes of t_{ORN} exposures for the same SNR. Also assume we are allowing readout noise to increase the sky noise by 5%, so p=.05. Then here are the resultant total exposures of shorter frames required to equal the SNR of the t_{ORN} exposures:

f	**N_2 / N_1**	**Exposure time**	**Number**	**Total Time**
1	1	10 min.	5	50 min.
2	2.186	5 min.	11	55 min.
3	3.558	3.3 min.	18	59 min.
4	5.115	2.5 min.	26	65 min.
5	6.86	2 min.	34	68 min.

This table indicates a number of points:

1. We get the biggest bang, i.e. increase in the number of samples, for f=2
2. Shorter and shorter exposures require increasing number of additional frames and therefore more total time to maintain the SNR. This is not surprising in that both readout noise and sky noise must be reduced.

The key point is there is no difference in the random noise SNR between the various values for f, as long as the appropriate number of exposures (N_2/N_1) for a given f value are taken.

We can examine the converse situation. With the same exposure assumptions as above, we can see how SNR is gain or lost if we keep the total exposure time constant and vary the sub-exposure length.

f	**Exposure time**	**Number**	**SNR Gain/-Loss**
0.2	50 min.	1	3.9%
0.4	25 min.	2	2.9%
1	10 min.	5	0%
2	5 min.	10	-4.3%
3	3.33 min.	15	-8.2%
4	2.5 min.	20	-11.6%
5	2 min.	25	-14.6%

This illustrates the impact of going to either side of the t_{ORN} exposure time. Using longer, fewer exposures achieves a slight gain in SNR but with less frames. Using shorter, more exposures has SNR losses shown above. But for f=2 and extending to f=3, a few more sub-exposures of 1 or 3 respectively, the SNR be maintained. However we will have more total frames.

There are a number of advantages of having more frames to deal with. First, we can use combination routines that are more sophisticated than a simple mean or median. MM-clip in Mira, Sigma from Ray Gralak to name a few, when coupled with dithering[1], enable excellent artifact rejection, cosmic ray rejection and even hot and cold pixel rejection. This results in a much smoother image.

Methods of combining sub-exposures

Let's look at some SNR relationships for various combining algorithms.

For a mean or average combine,

(13) $$SNR \propto \sqrt{N}$$

A mean combine reduces non-random noise only to the extent it does not occur at the same pixel for each frame and then the reduction in non-random noise is only to scale it by N.

For a median combine,

(14) $$SNR \propto \sqrt{\frac{2(N-1)}{\pi}}$$

Median combine provides better reduction of non-random noise, but at a fixed penalty of random SNR loss. MM-Clip is a combine routine which throws out the minimum and maximum value for each pixel in the set, provides excellent artifact reduction with less random SNR loss than a median combine. This is an example of an "outlier-rejection algorithm. It rejects data that is far removed from the nominal data trend. For a MM Clip,

---

[1] See www.hiddenloft.com/notes/dithering/htm for a discussion on dithering.

$$(15) \qquad SNR \propto \sqrt{N-2}$$

So, for random processes, the mean combine gives the best random noise reduction, but non-random noise reduction is minimal. For median combine, some non-random noise reduction is achieved but at a SNR loss of

$$(16) \qquad Loss_{Median} = 1 - \sqrt{\frac{2(N-1)}{\pi N}}$$

However, for the MM-Clip combine, the SNR loss is given by

$$(17) \qquad Loss_{MM-Clip} = 1 - \sqrt{\frac{N-2}{N}}$$

The point at which the MM-Clip loss equals that of the median can be developed by setting equation 16 equal to 17 and solving for n. The resultant number of frames is

$$(18) \qquad N = \frac{1}{1 - \frac{2}{\pi}} = 2.75$$

Thus, for N > 3, the SNR of MM-Clip will exceed that of the median, providing good artifact reduction and improved SNR, as compared to a median combine. Here are some random noise SNR losses relative to a mean combine as a function of the number of frames:

N	MM-Clip SNR Loss	Median SNR Loss
3	42%	35%
4	29%	31%
6	18%	27%
9	12%	25%
12	9%	24%
15	7%	23%
18	6%	22%

Thus, for any number of sub-exposures greater than 4, excessive noise is added by a median combine when compared to a MM-Clip combine. In essence, you are throwing away SNR with a median combine. Further, the larger the number of sub-exposures, the less the SNR loss with a MM-Clip combine. On the other hand, a median combine approaches a SNR loss of 20% and never improves with additional sub-exposures.

In my experience, I have found Sigma and MM-Clip combine give comparable SNR's, when the data is properly acquired. Sometimes one is better than the other. For example, if insufficient dithering is used, a MM-Clip algorithm will not be as good as a standard-deviation masking routine like Sigma. In any case, there is sufficient data to support the use of such combining algorithms in reducing non-random noise. These combining algorithms are most effective when there are a sufficient number of data points, i.e. sub-exposures, to combine.

Recommendations

To reduce both random and non-random noise, the following steps are recommended:

1. The minimum sub-exposure time should be chosen to be one-half of the "overwhelm read noise time" ($t_{ORN}/2$) to enable more frames to be taken in the defined period of time available.
2. A "statistically significant" number of frames should be taken, at least for the luminance data, to allow successful non-random artifact reduction. If your prior practice was to take n sub-exposures of t_{ORN} duration, instead take 2n+1 sub-exposures of $t_{ORN}/2$ duration.
3. Dither your sub-frames with sufficient movement such that non-random noise occurs in different locations on the registered sub-frames and therefore can be greatly reduced.
4. Combine your data using appropriate outlier rejection combination algorithms. Examples of such algorithms are MM-Clip and the many sigma-reject techniques in CCDStack, Mira or Ray Gralak's unique Sigma program.

Advantages and Disadvantages of this approach

As with any technique there are some benefits and compromises with this approach. The main advantages are:

- More data points for outlier rejecting combine algorithms allowing smoother images.
- Less star saturation leading to reduced blooms and better star colors
- Shorter exposures lead to the possibility of unguided imaging
- Less data loss due to satellites, airplanes and other accidents

The disadvantages are:

- More files to deal with during data reduction, registering and combining
- More time during acquisition not only for the additional frames required but also for downloading, dithering, guiding restarting.
- A larger number of dark frames required for the master dark, albeit with the same total time for reasonable camera specifications and operating conditions.

With high speed downloads, a lot of these issues are mitigated and it is believed the advantages far outweigh the disadvantages.

Summary and Conclusions

An analysis of SNR has been presented considering two of the main sources of noise in images – random noise and non-random noise. Previous sub-exposure strategies have concentrated exclusively on reduction of the former and not considered the latter. By a modest increase in total exposure time and a little more processing complexity, a method is described which addresses both noise sources. The method consists of 2n+1 sub-exposures with t_{ORN} /2 exposure duration, coupled dithered image acquisition and outlier-rejection combining algorithms. It is felt that this approach will result in an overall higher total data quality, when considering the noise contribution of both random and non-random noise sources.